HOW WE ‘KNOW’: MAKING DISCOVERIES IN MODERN PHYSICS

Lecture 3: probability and inference

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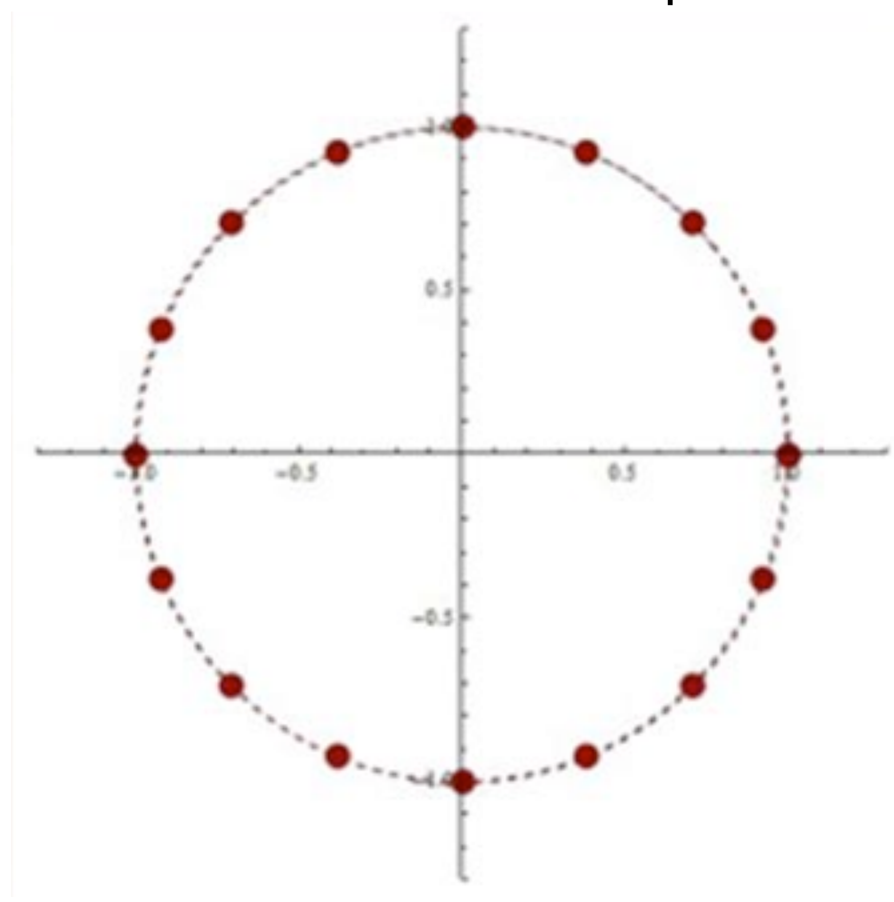

This lecture

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- Gravity and gravitational waves

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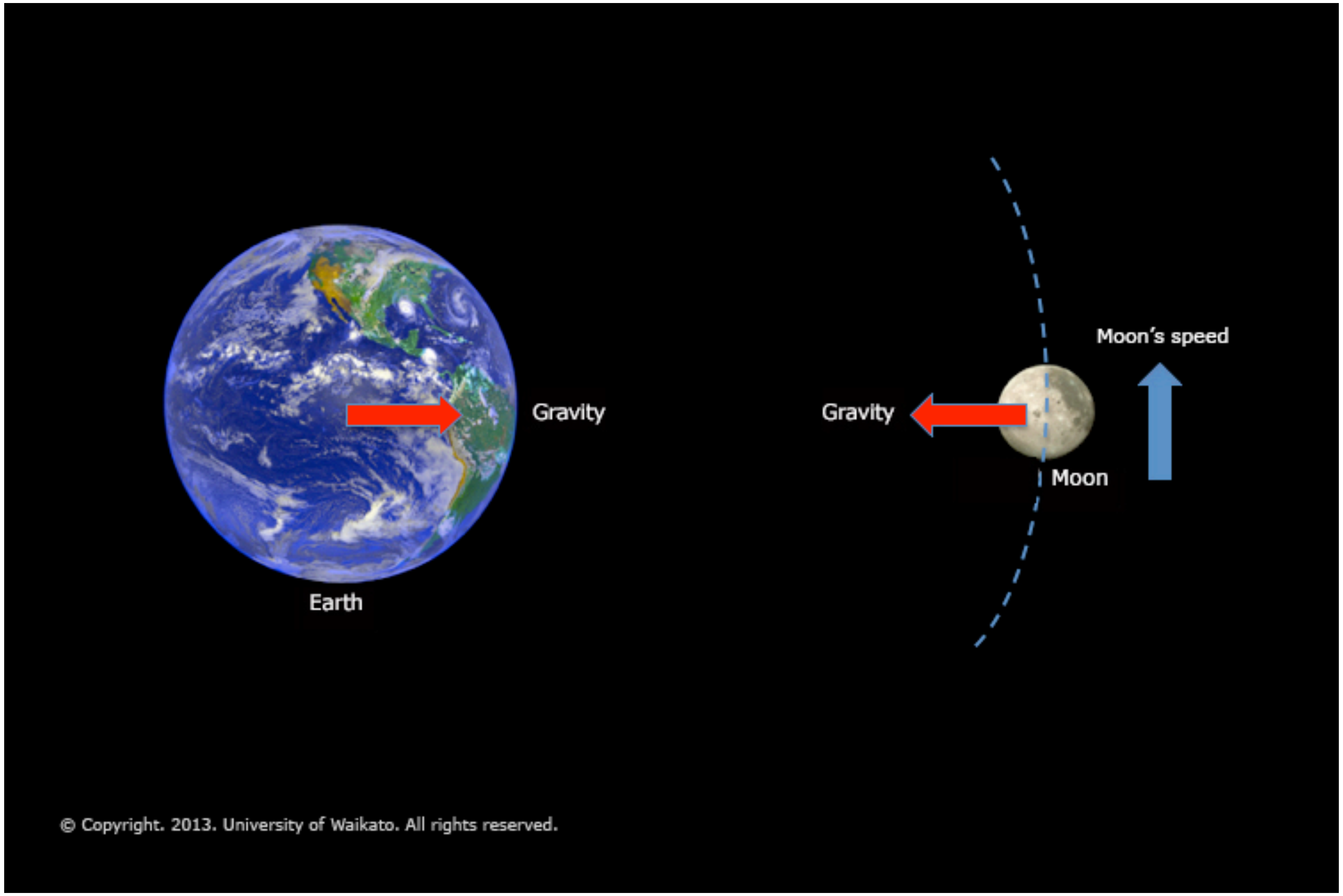
- Models

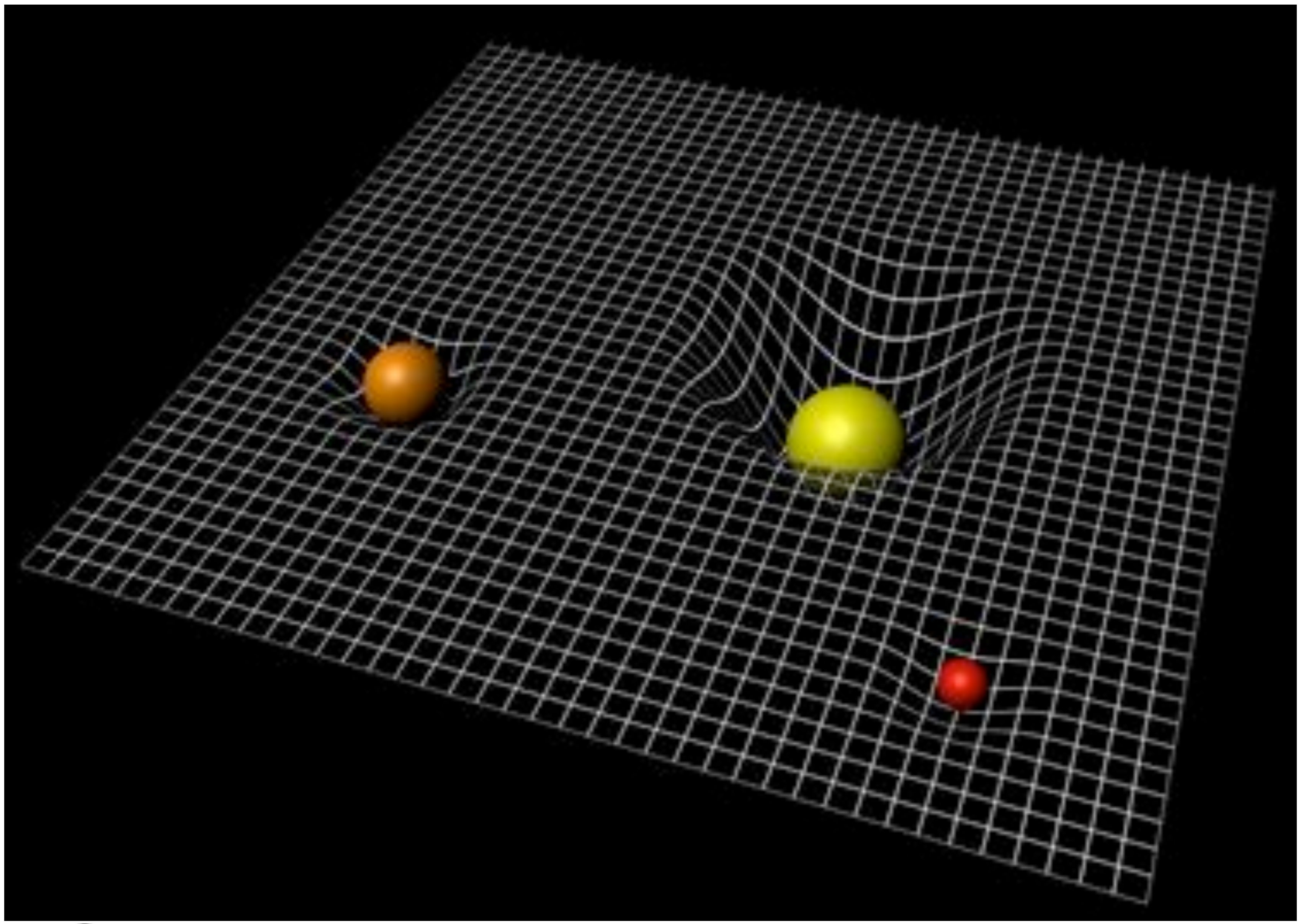
Einstein’s Gravity

Newton: Gravity is an attractive force between two objects with mass



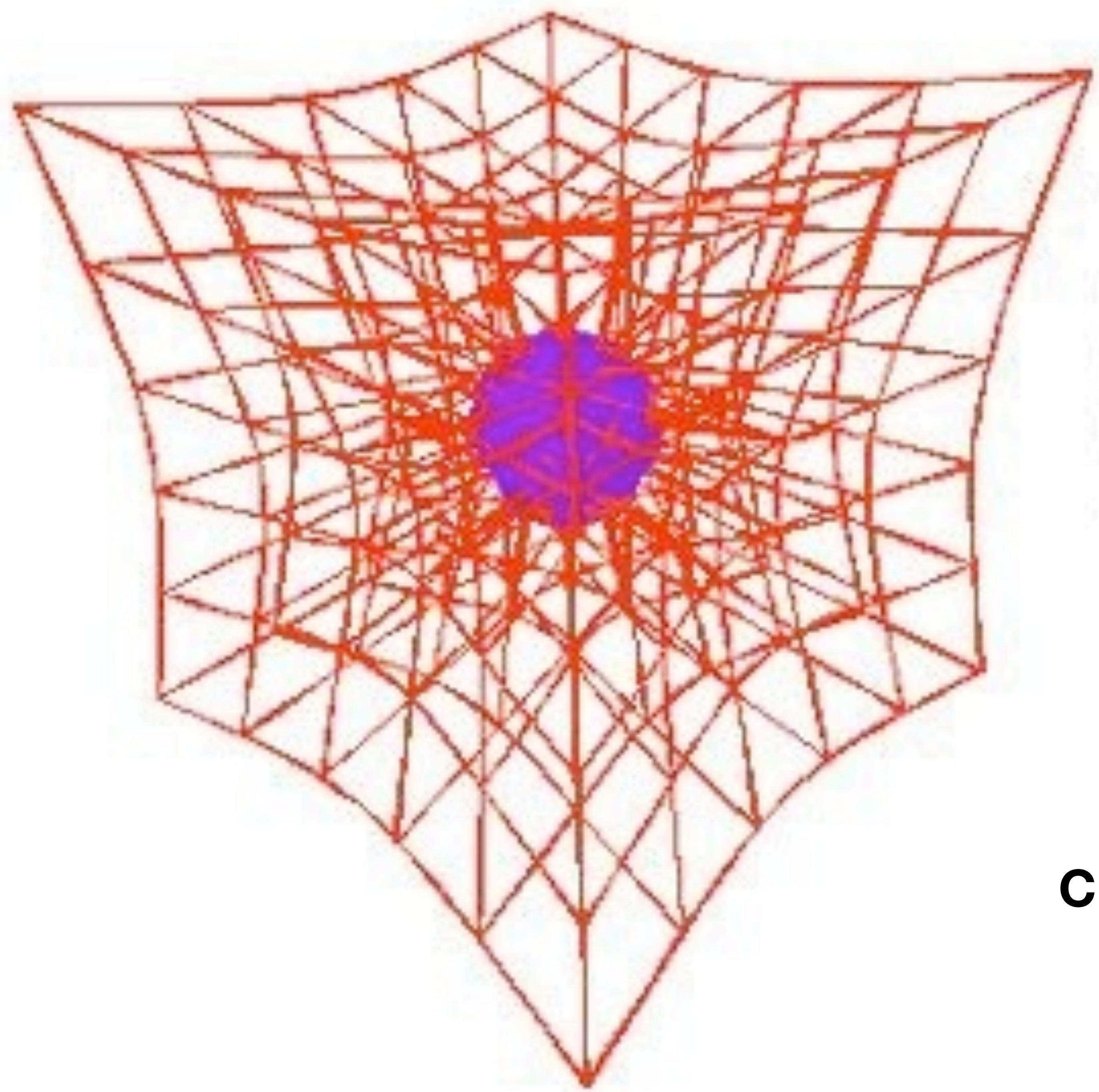
Einstein's Gravity

Einstein: Gravity is acceleration in curved spacetime
"Matter tells spacetime how to curve, spacetime tells matter how to move"



Rule: can only move along the 'sheet' of spacetime in 'straight lines'
Matter curves spacetime - straight line becomes curved = acceleration!

Einstein's Gravity

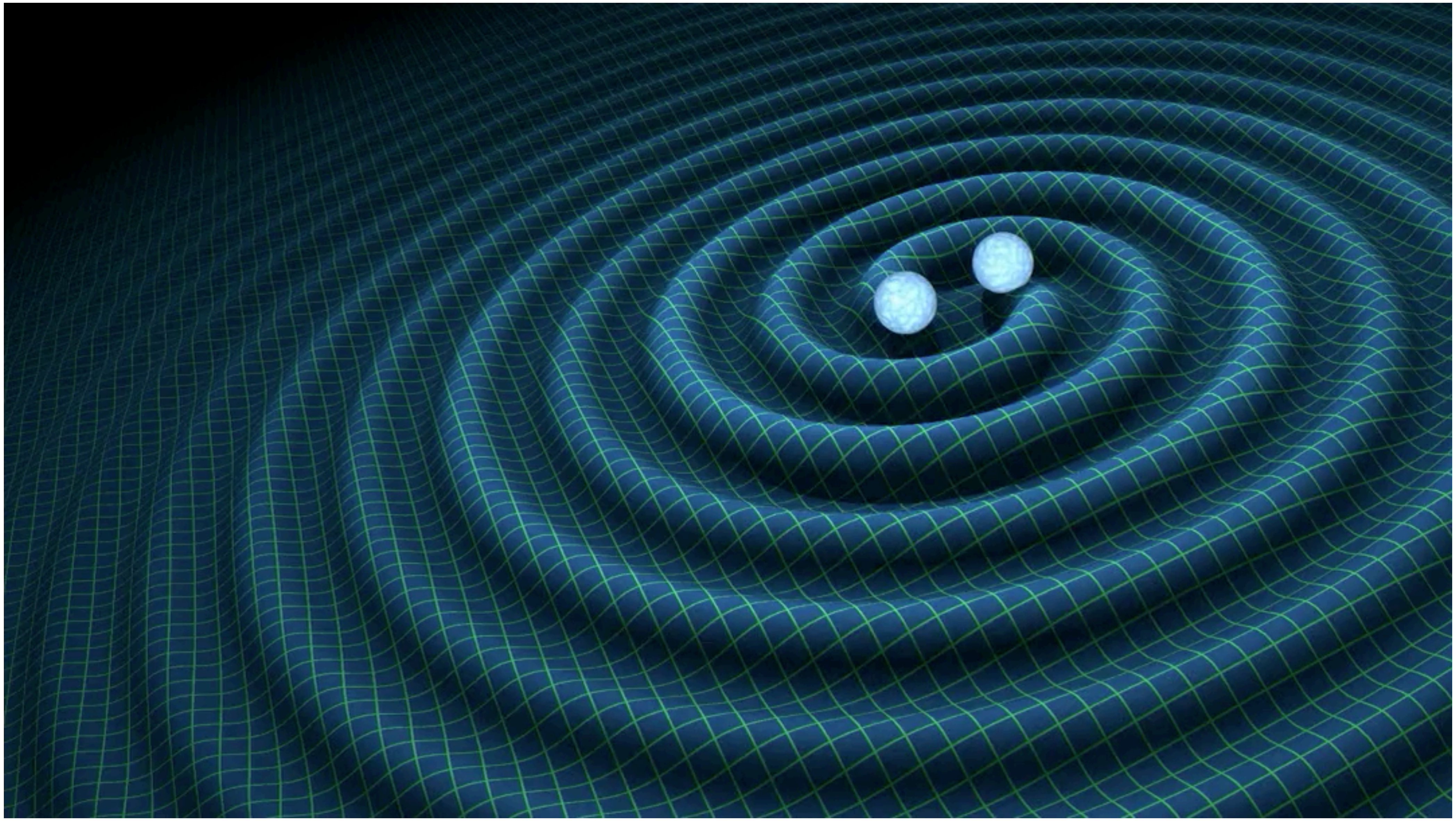


This three-dimensional grid gives a better idea of what curved space-time might look like than the two-dimensional analogies do.

Clocks closer to the mass run slower than those further away

Gravitational Waves

Einstein noted that if masses accelerate, they cause spacetime to ‘ripple’



Spacetime is very stiff - these ripples are tiny

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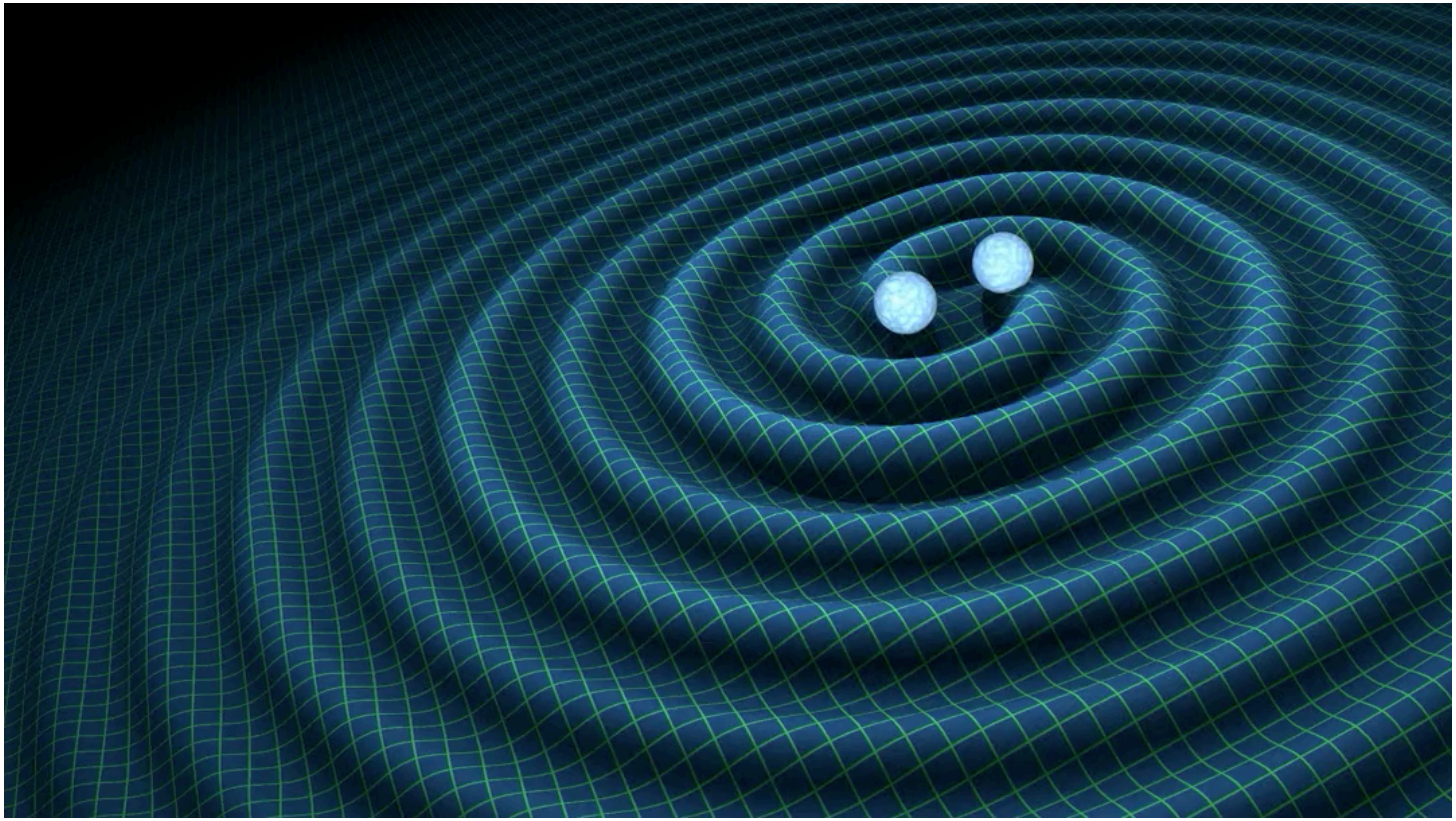

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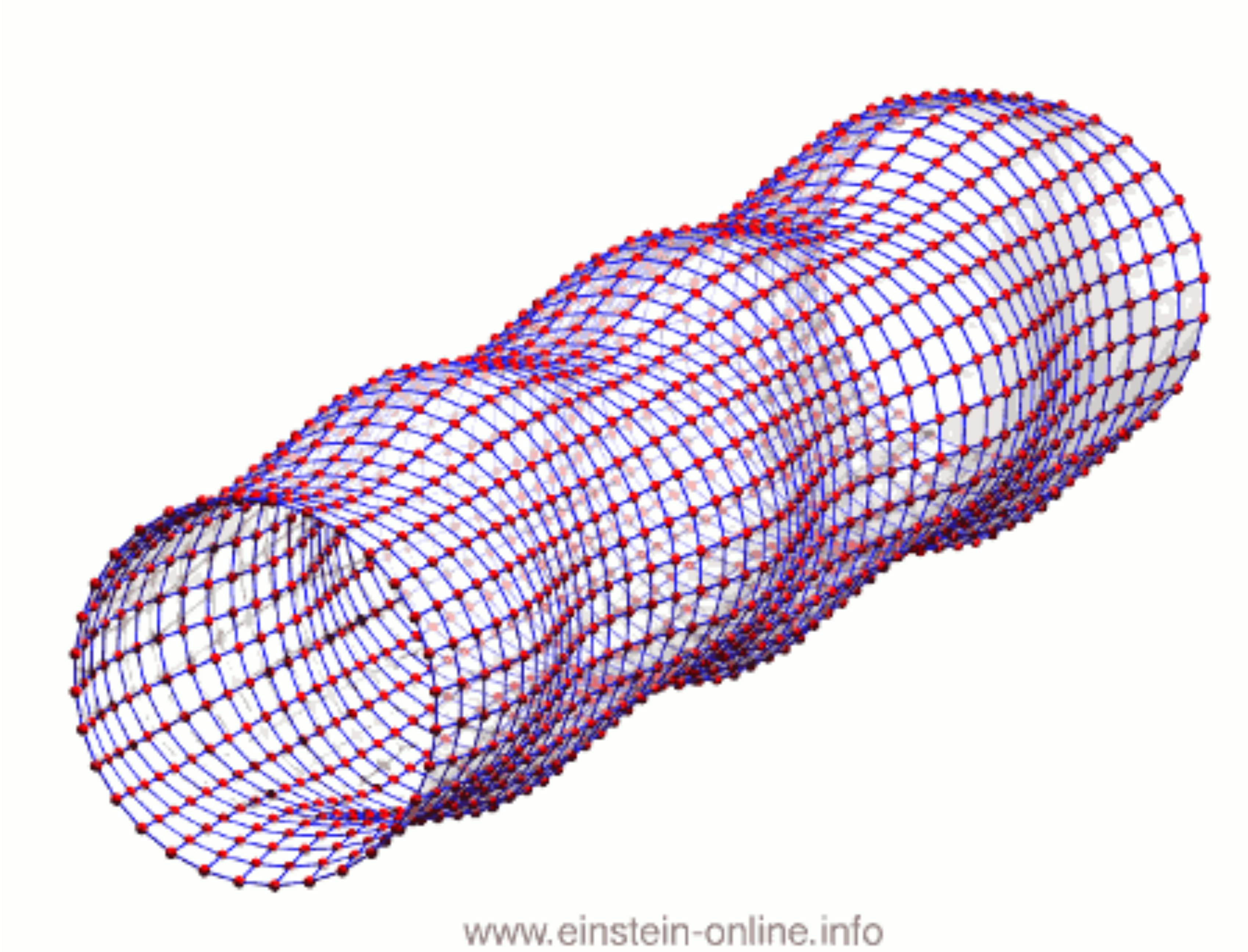
Gravitational Waves

This is a simplification to view gravitational waves as ripples in a pond



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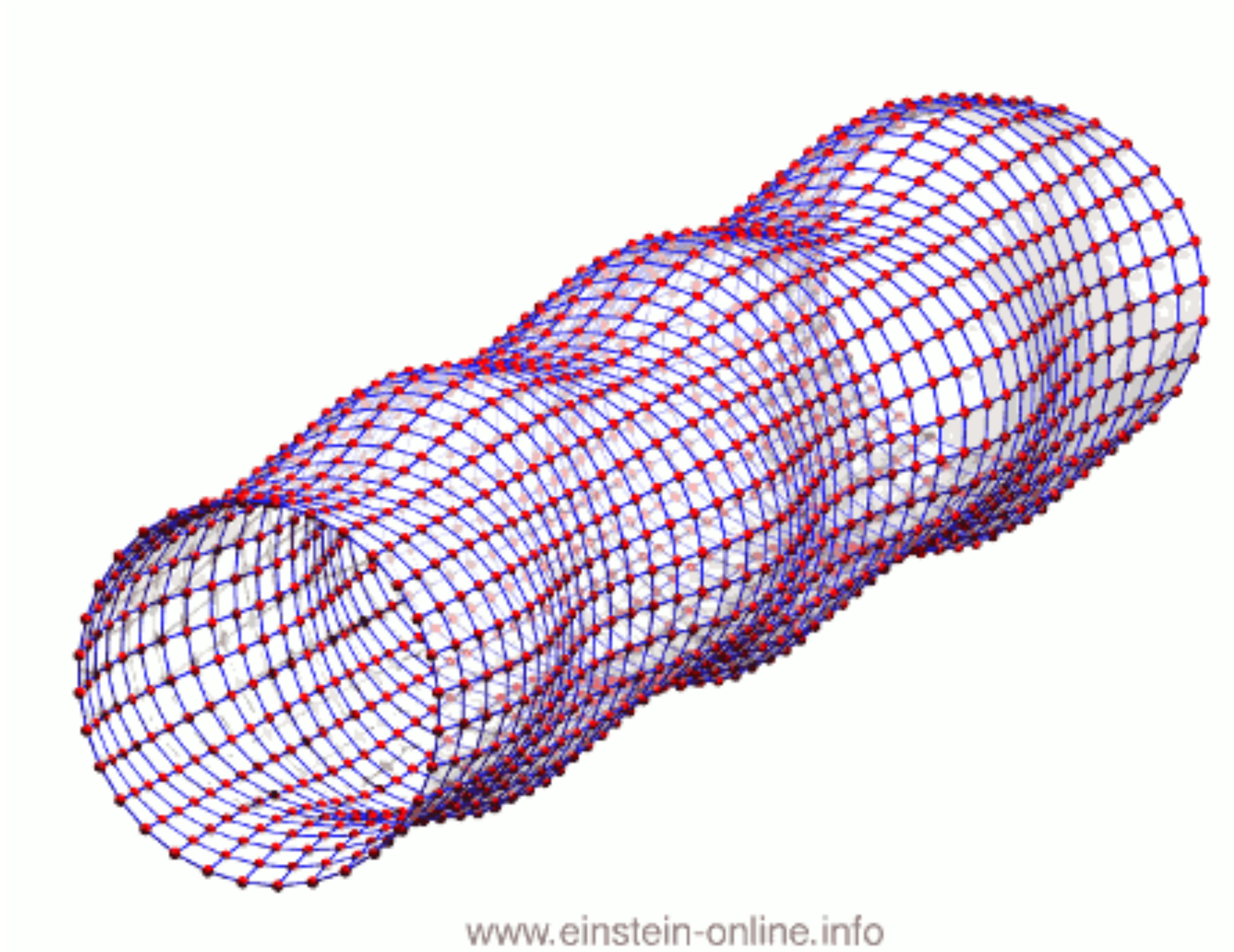
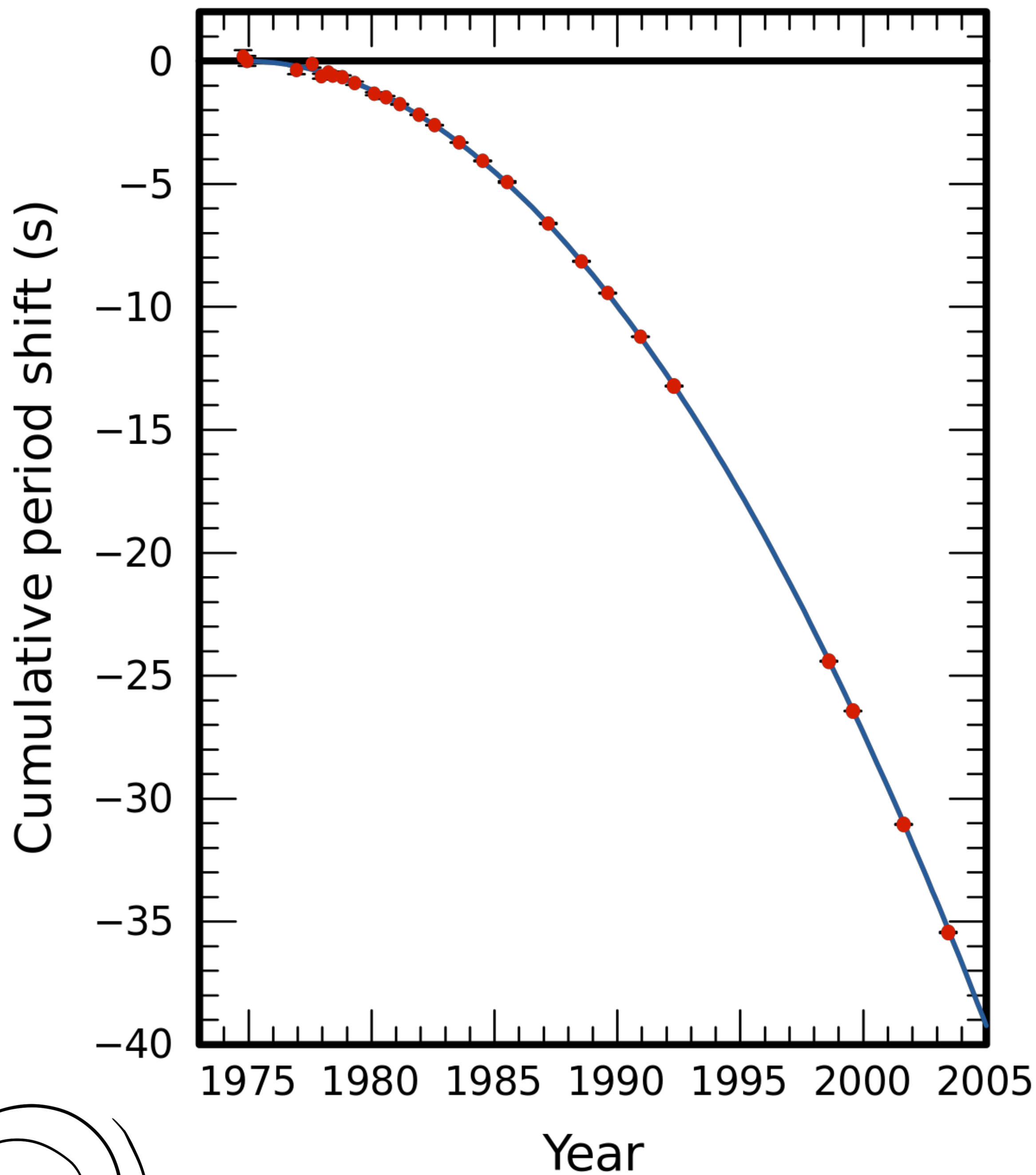


Discovery

Emission of gravitational waves results in energy being lost from the system



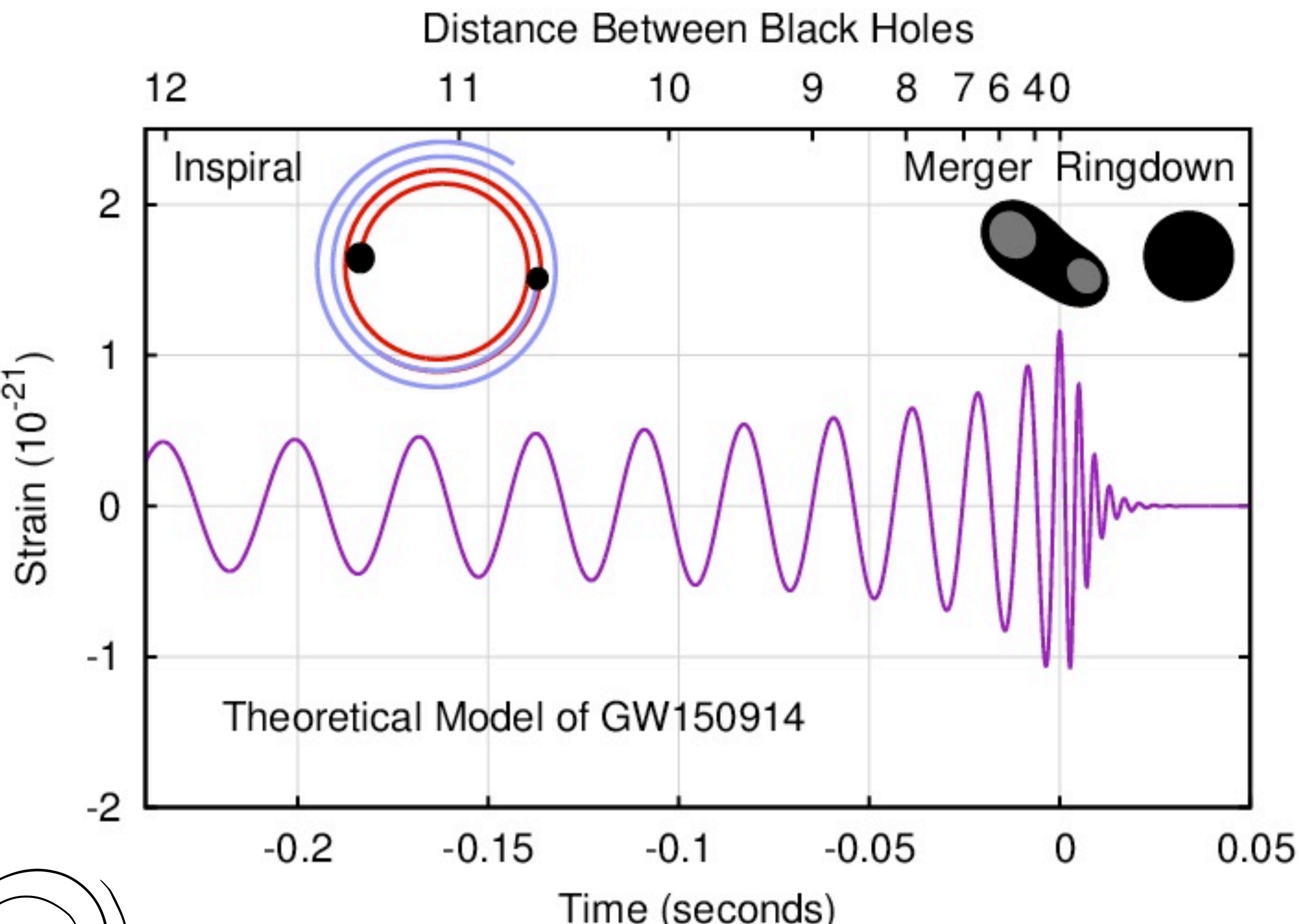
Most stars in the Universe come in pairs. Eventually these binary systems will evolve to become stellar remnants. Some may become binary neutron stars, or binary black holes

We observe many neutron star binaries in our galaxy as they emit pulses of radio waves like lighthouses

GWs were indirectly discovered by observing the decay of the orbits of a double pulsar by Hulse and Taylor

Discovery

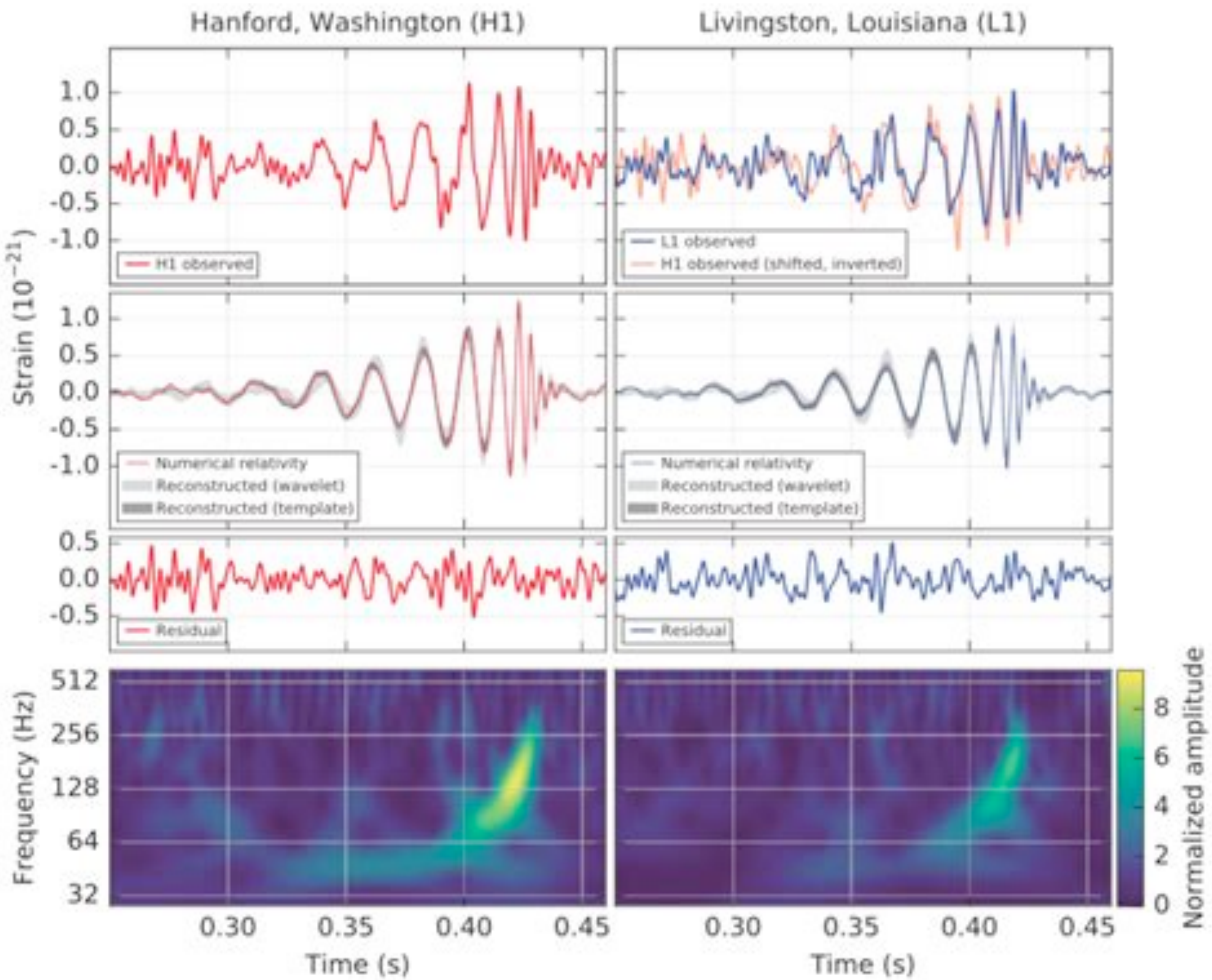
To detect gravitational waves directly, we need to measure the changes in spacetime caused by the gravitational wave



We understand a lot of the physics needed to derive how we expect the change in length of spacetime at Earth to behave when black holes (or neutron stars) merge

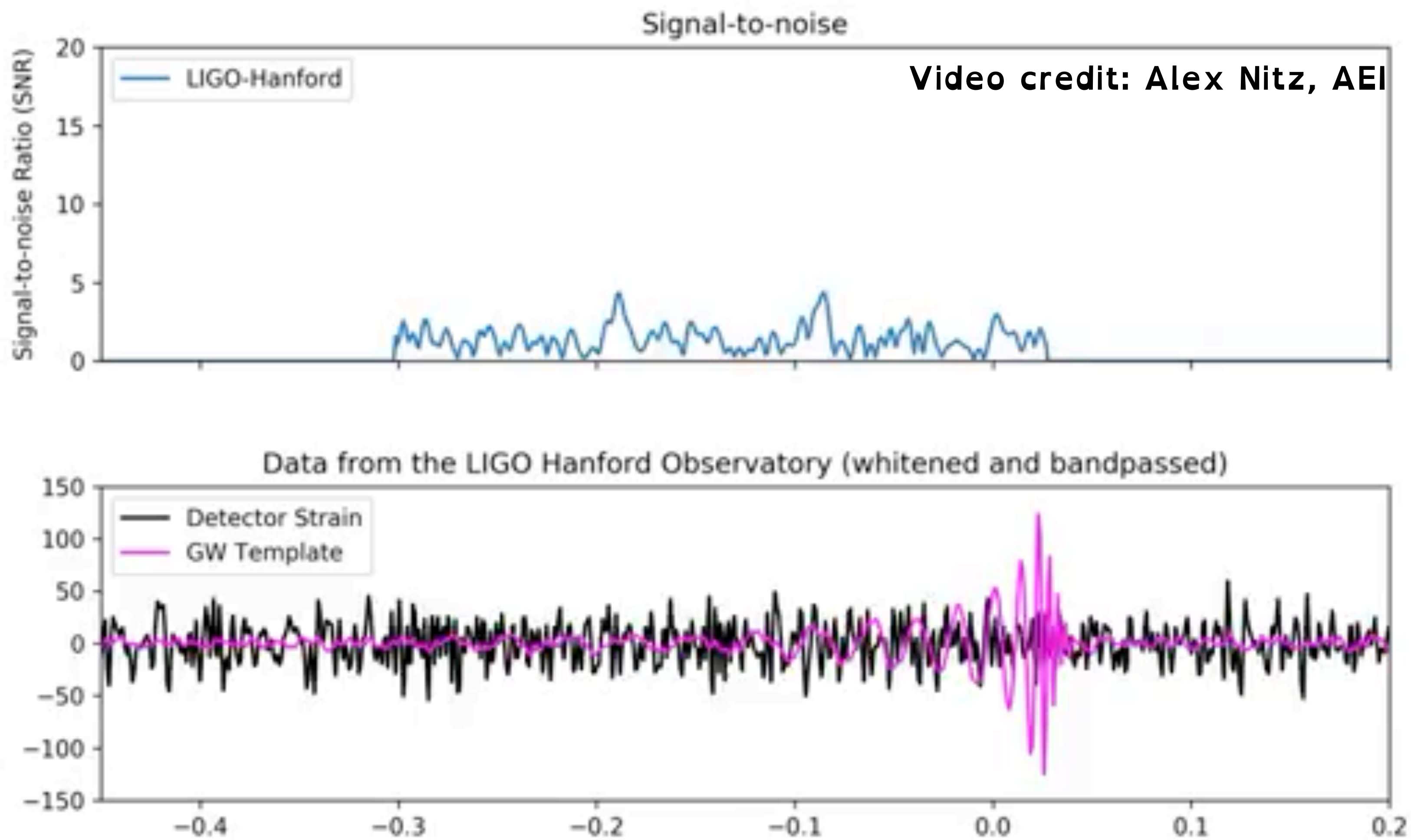
Discovery

If we collect data, we can see what parameters that describe the physical properties of the system best fit the data (the waveform changes when these parameters change)



Discovery

Several steps: first identify possible signals and then use that information to accurately estimate exactly what produced them



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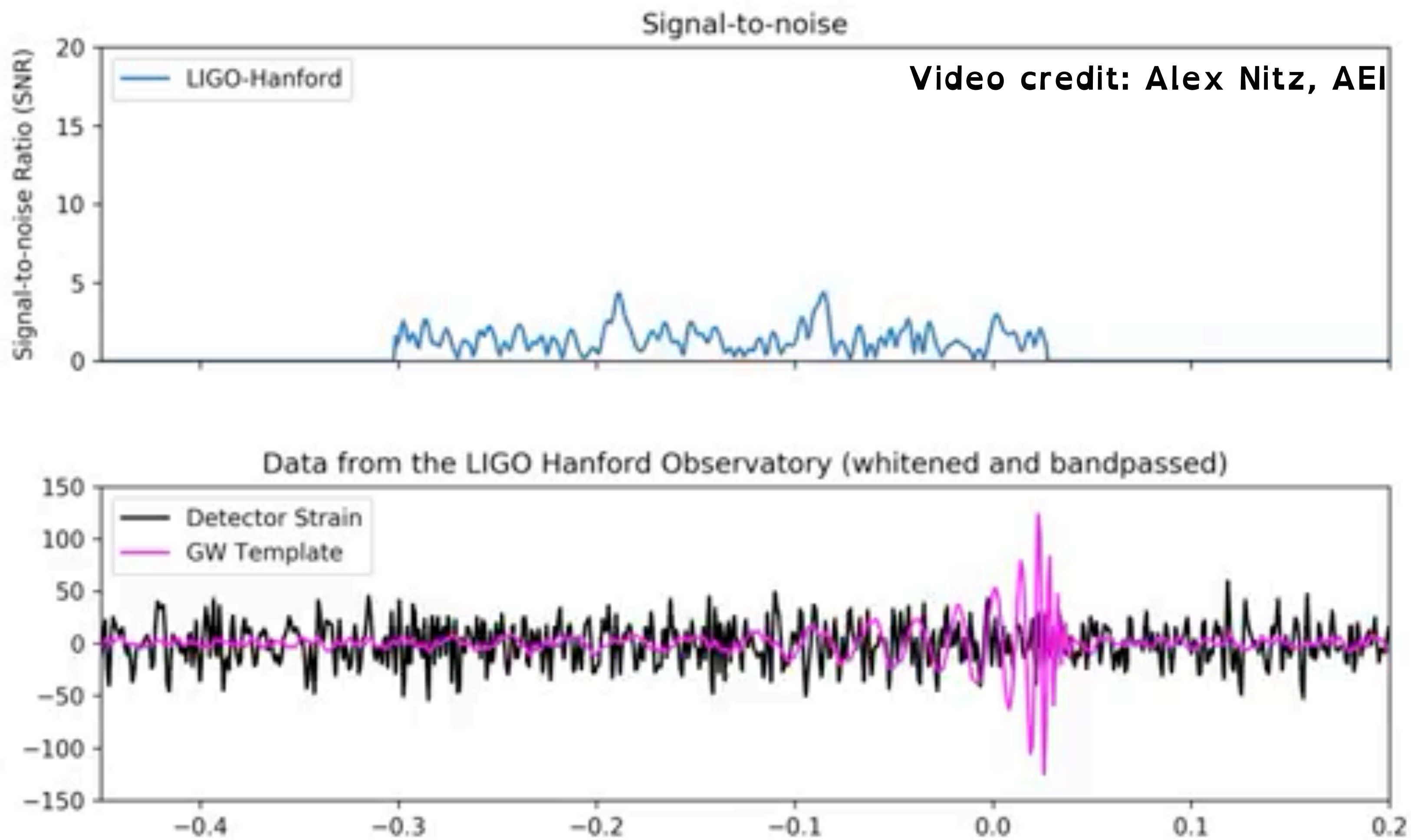

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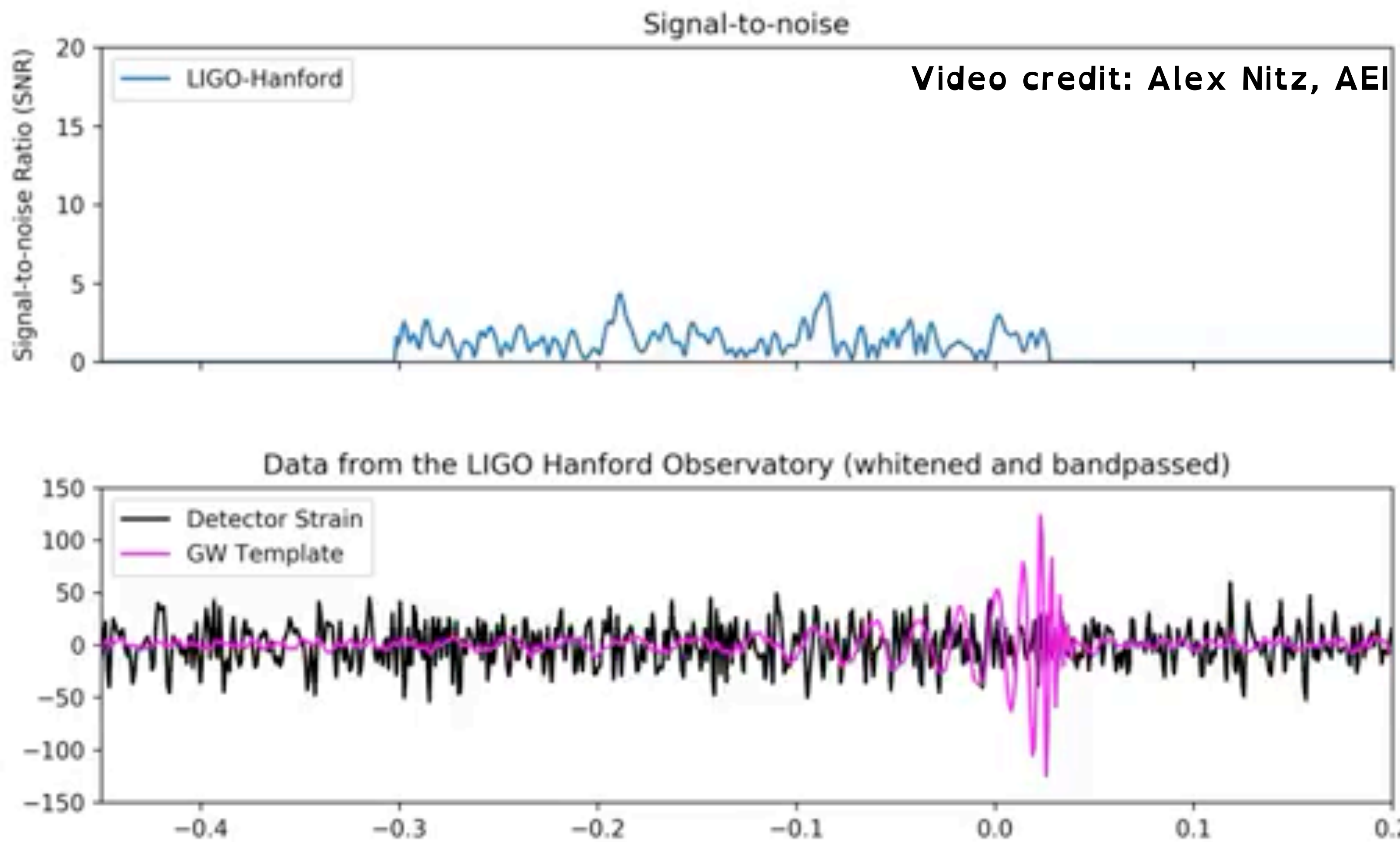

Do this with ~ a million different waveforms in real time

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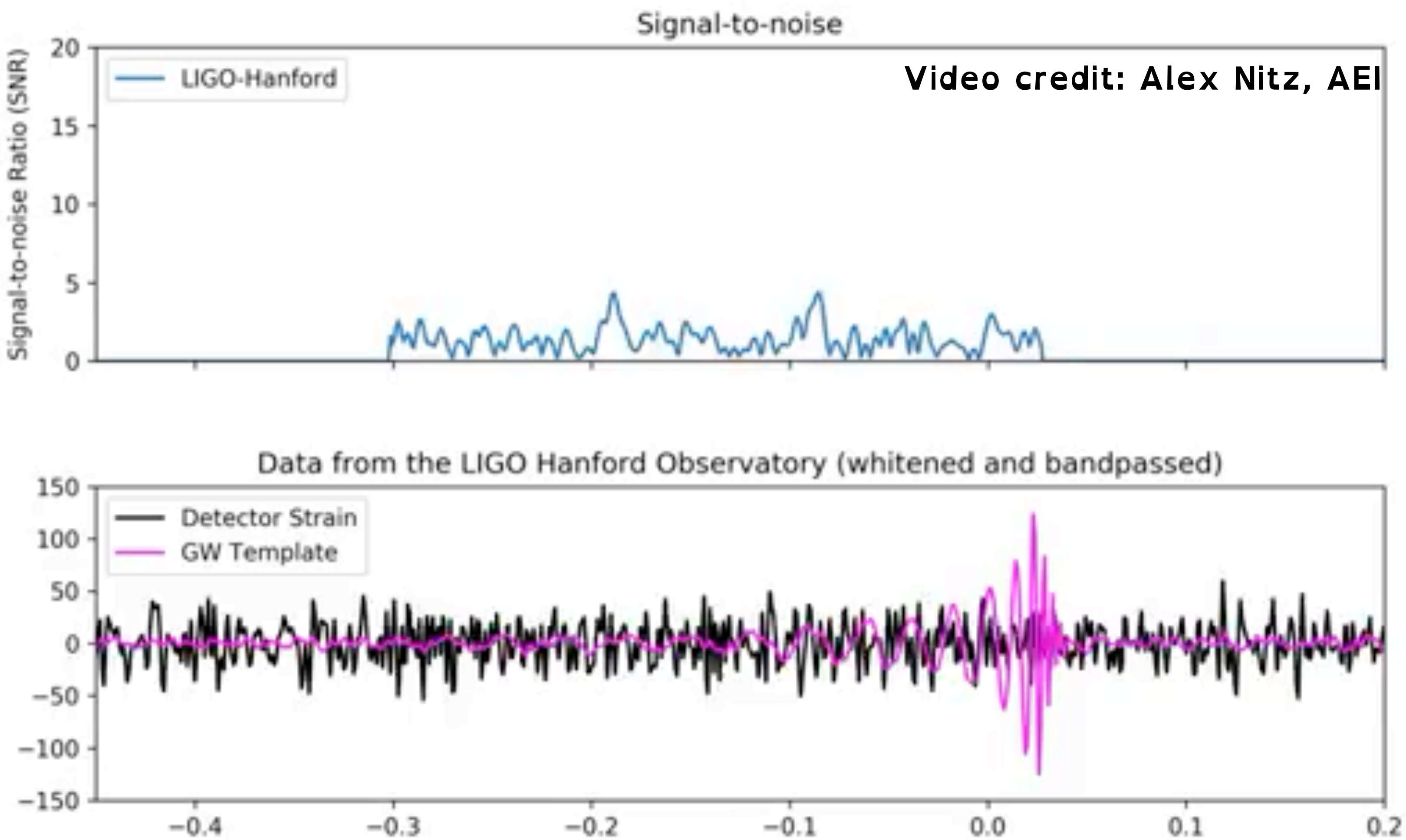


Do this with ~ a million different waveforms in real time

Many checks are made to ensure we haven't actually mistaken noise for a gravitational wave

Bayes Theorem

Probability measures a degree of belief in a statement or event. Bayes Theorem links the degree of belief in a statement before and after making a measurement

What we initially believe about A

What we believe about A now we know B is true

$$P(A \mid B) = \frac{P(B \mid A)\; P(A)}{P(B)}$$

What support B provides for A

Thus we can start to make statements like ‘there is a 95% chance that the value of x measured from this data lies between y and z’

(note the difference to the frequentist statement which is: If you repeat the experiment, 95% of the time, the value you measure of x would be between y and z)

Bayes Theorem

Rev. Thomas Bayes published the famous equation in 1763.

Laplace also came up with the same arguments in 1774

The version that is most commonly used today is Laplace's version, which views the conditional probabilities as 'the probability that x is true' rather than a proportion of outcomes

Example: weighted coin

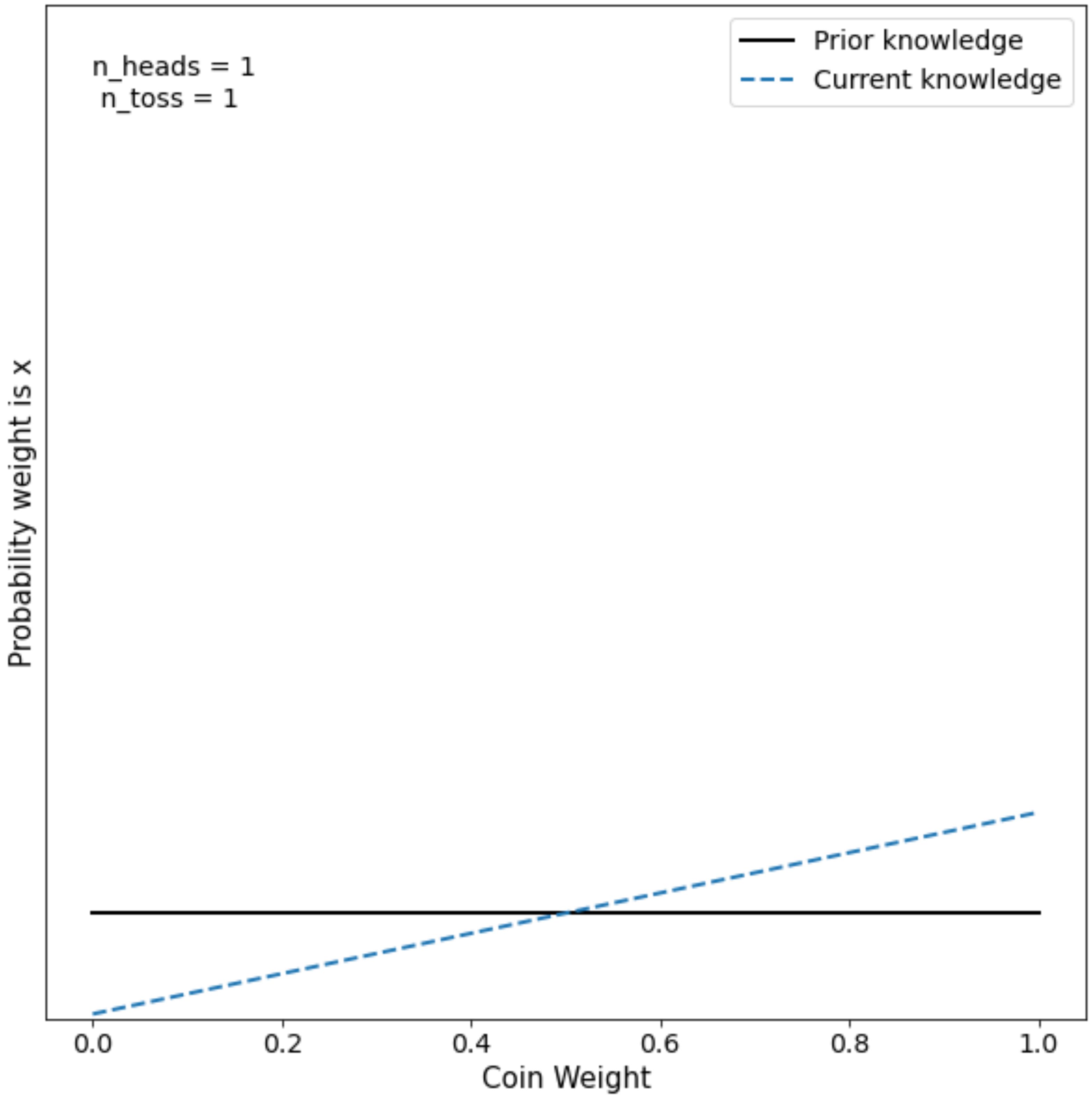
Want to calculate the weighing of an unfair coin

Ingredients

- Data: results of coin tosses
- Prior: we don't know anything - so any weighting is equally likely (uniform prior)
- Likelihood: binomial (the result can be one or the other - this is just a formula you look up. More about likelihoods next time)

In reality we can just collect all the data and we don't have to update every single step, but this example shows how updating gives us more information on an unknown parameter in our model

Example: weighted coin



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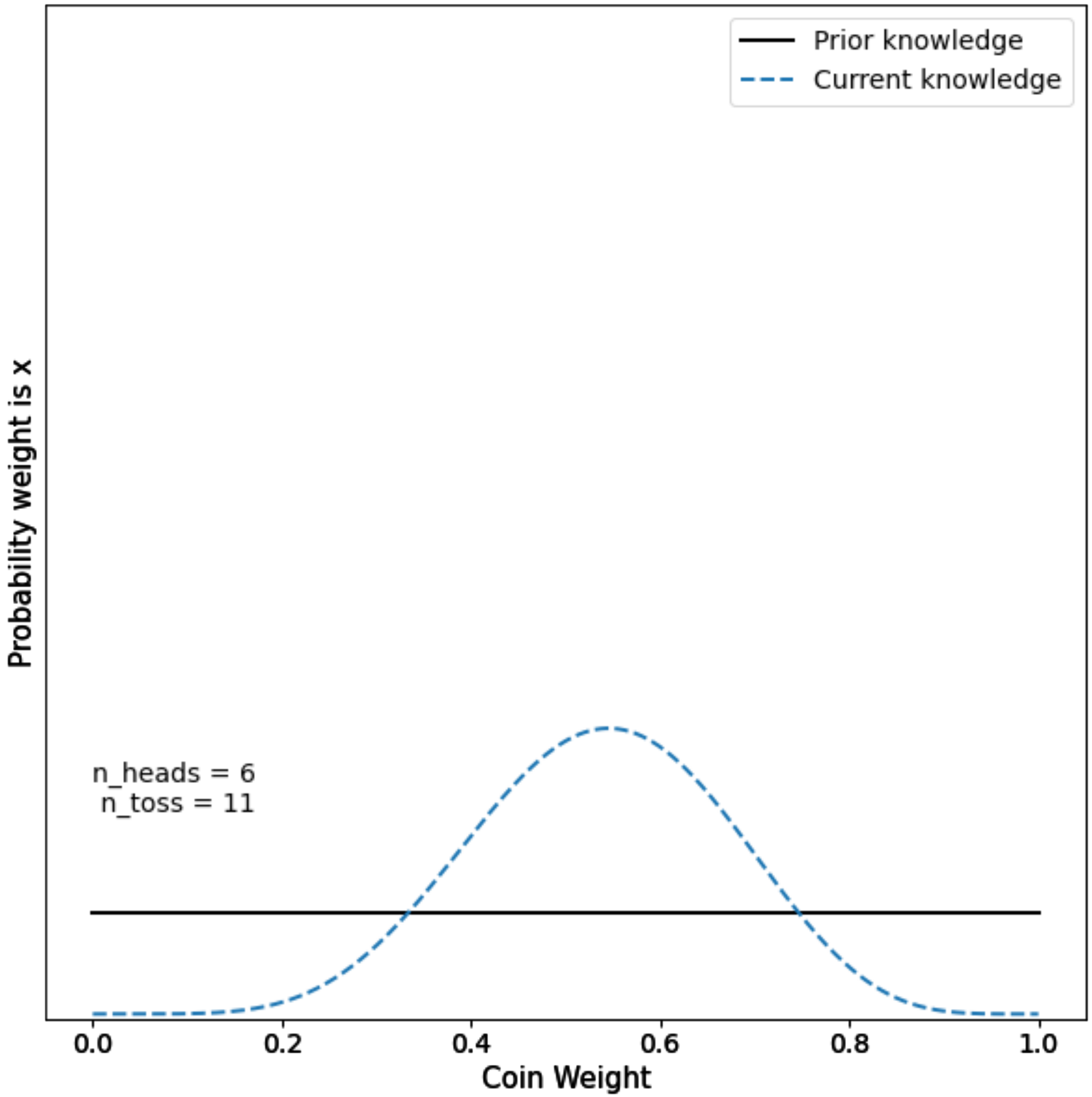

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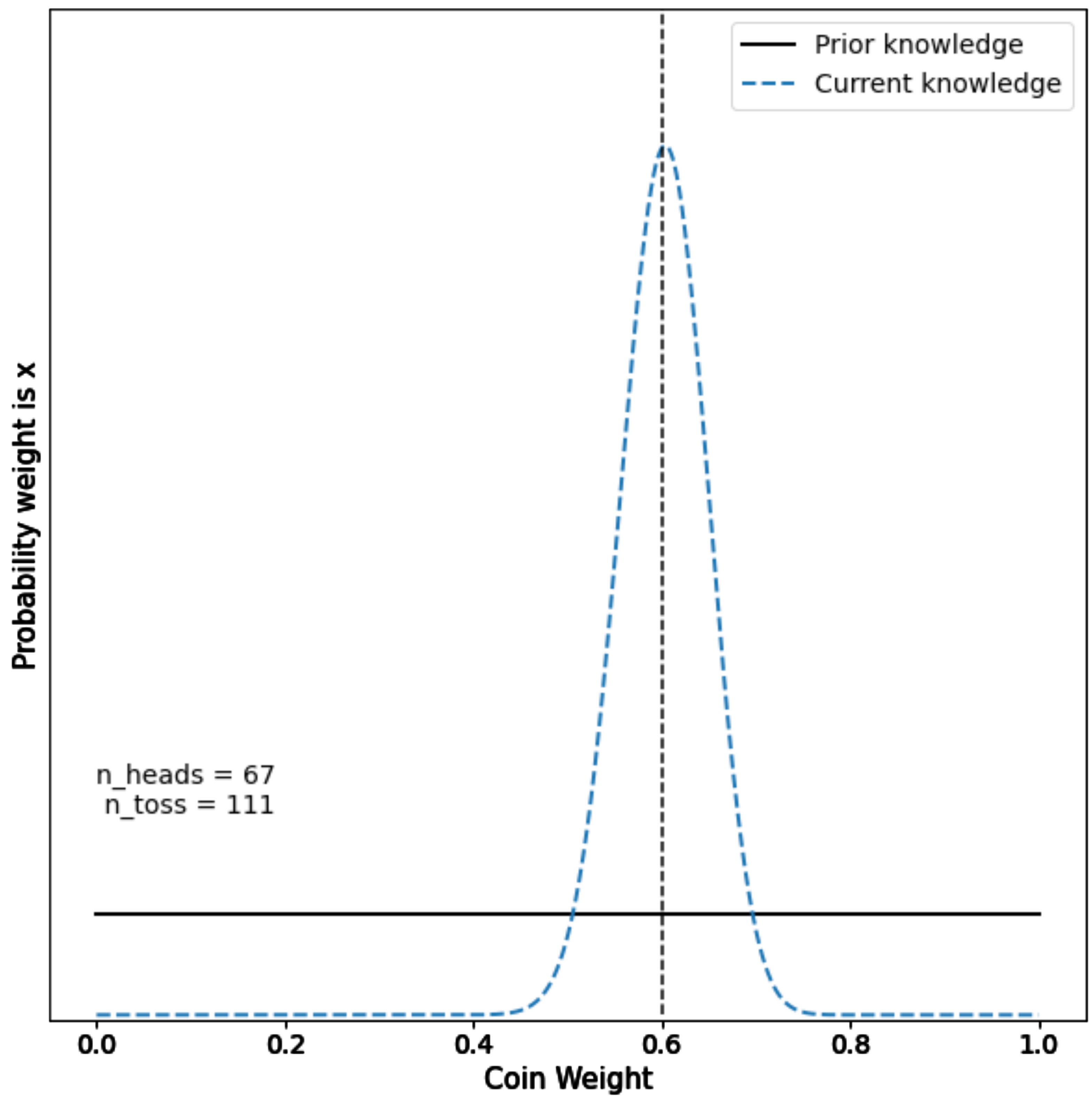

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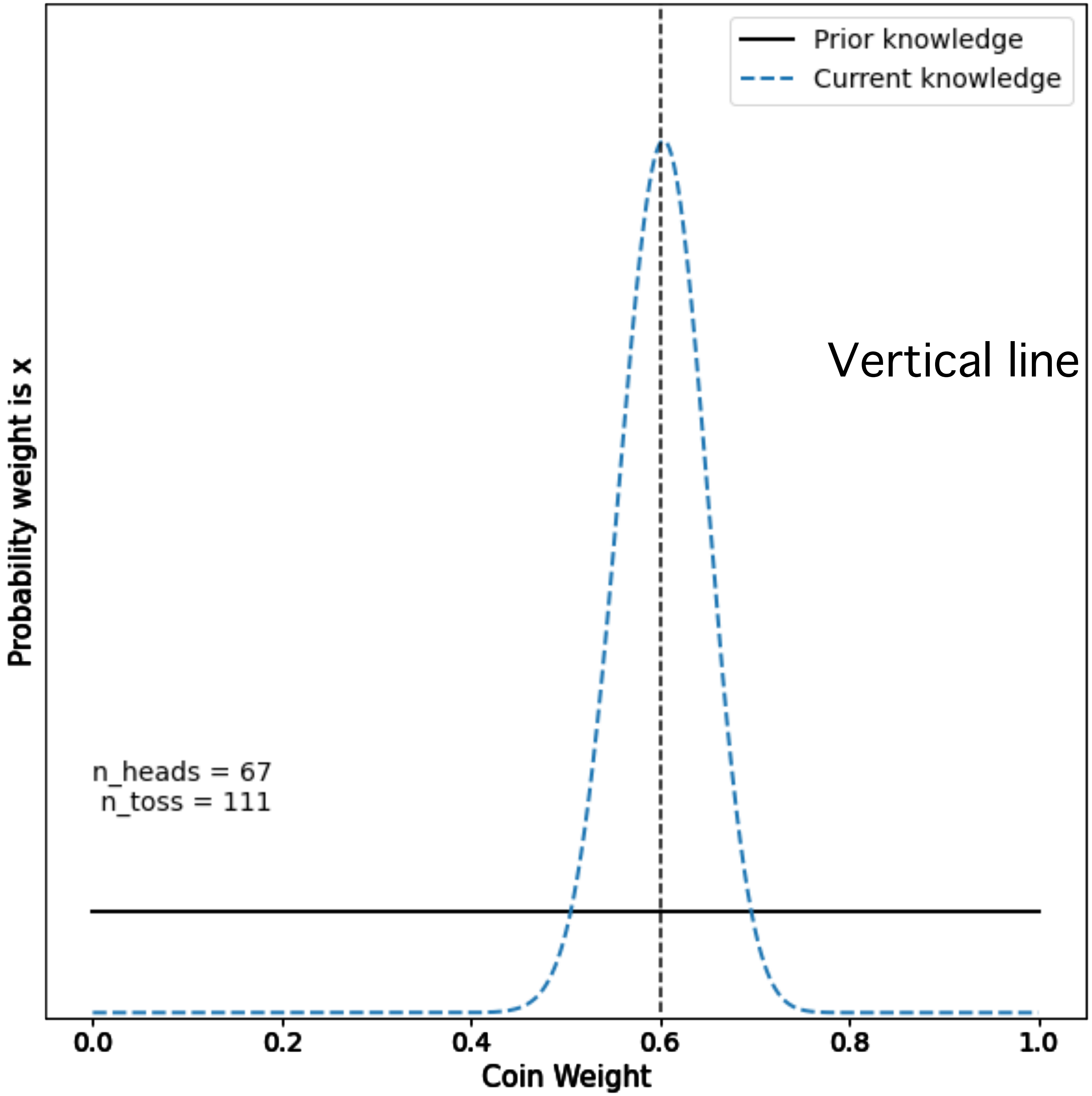


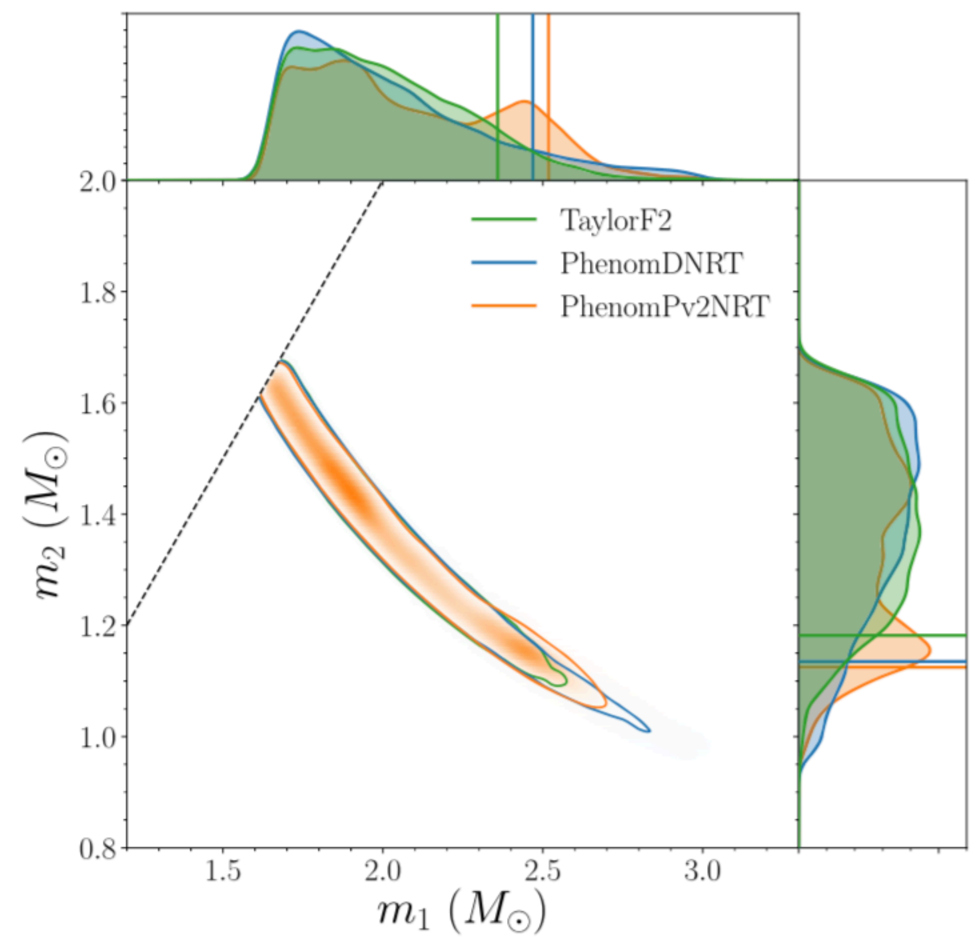
Vertical line - the true value!

Parameter Estimation

Chirp signal of a gravitational wave can be predicted from theory - depends on astrophysical stuff

We want to know: masses of black holes in the universe, black hole spin, distance to black holes (15 parameters in all), but this is hard to formulate in the frequentist world

Parameters also depend on one another - complicated problem, can take months of computing time!



Example: estimating the probability that the components of a neutron star merger (GW190425) has a particular individual component mass

When models go bad

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- Next lecture: let’s talk about models and assumptions - what is a good one, and what happens when models go bad